**Mathematical Methods in Engineering and Applied Science**

**Problem Set 4**.

By Kovalev Vyacheslav.

1. Explain:
2. Why is not singular when matrix has independent columns;

Let , it means that A has independent rows, and are independent too.   
Let , where may be expressed as a combination of columns : *,* among which r independent columns. In other words, consist of r independent columns , B is full rank => B is non-singular.

1. Why and have the same nullspace.

Let’s A is m by n matrix.

Let’s prove that if

If

But for it’s true that:   
and for .  
Since (similar to prev. task) But we found that =>

1. A plane in is given by the equation .
2. Identify two orthonormal vectors and that span the plane.

Let’s choose any two vectors of the plane

Let’s ;

is supposed to be orthogonal to : After normalization:

1. Find a projector matrix that projects any vector from to the plane and a projector that projects any vector to the direction normal to the plane.

From prev. task:

1. Using these projectors find the unit normal to the plane and verify that it agrees with a normal found by calculus methods (that use the gradient).

Normal of plane is

Otherwise: choose vector that doesn’t satisfy plane equation: [2,0,0] for instance and act on it by

And normalize it:

1. Let where
2. Find the orthogonal projector on .

Determine ;

1. Find the kernel (nullspace) and range (column space) of .

– plane projector, , that’s why column space is any 2 independent vectors

For null space: Let columns of , ;

1. Find which is closest in 2-norm to the vector.

Suppose ; in other words consists of ,perp to M and no perp to M.

To make ,

1. he following problems look at tests of positive definiteness.
2. Using the determinant test, find and that make the following matrices positive definite:

Leading determinants must be positive:

For second matrix:

impossible to make B positive defined

1. positive definite matrix cannot have a zero (or a negative number) on its main diagonal. Show that the matrix

Let’s ; then ;

1. Matrix is positive definite. Explain why and determine the minimum value of, where and .

is positive defined because of leading determinants are positive.

The minimum is found from:

The approach to this task the similar as to solving equation: , where , because of A is positive defined. So, this equation describes a parabola with one extremum, and if then it is minimum.`

1. Explain these inequalities from the definition of the norms:

and deduce that

Let

Proved that

It is easy to show that , in the same way as why

Finally, .

– (using prev. condition.)

1. Compute by hand the norms and condition numbers of the following matrices:

Since is symmetric => from lectures ;

Therefor ;

Since is symmetric => from lectures ;

Therefor ;

Condition number of

For second matrix ;

Find e-values :

Therefor

And condition number: